MATH 54 - HINTS TO HOMEWORK 4

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Here are a couple of hints to Homework 4! Enjoy :)

Note: This homework has been trimmed a bit! For section 4.3, you only have to do 7 problems, and you can ignore 15 and for 22 you only need to do (a), (b), (c). Also, for $1, 5, 6$, you don't need to show your work! For section 4.4, for 15 and 16 you only need to do (a) and (b).

Section 4.1: Vector spaces and subspaces

Remember the three techniques of showing whether something is a vector space or not!

(1) Trick 1: Show it is not a vector space by finding an explicit property which does not hold
(2) Trick 2: Show it is a subspace of a (known) vector space
(3) Trick 3: Express it in the form $Span$ of some vectors.

Also, to show W is a subspace of V (and hence a vector space), you need to show 3 things:

(1) The $\mathbf{0}$ vector is in W
(2) If $\mathbf{u}$ and $\mathbf{v}$ are in W, then $\mathbf{u} + \mathbf{v}$ is in W
(3) If $\mathbf{u}$ is in W and c is any real number, then $c\mathbf{u}$ is in W

4.1.5. Yes

4.1.7. **NO!** The reason is the word *integer*. Remember that c can in general have *any* real value, including $\sqrt{2}$. So ask yourself: If p is in W, is $\sqrt{2}p$ in W? If W was a subspace, it would be!

4.1.9. Yes

4.1.15. No, the $\mathbf{0}$ vector is not in it!

4.1.17. Yes, express it as Span of 3 vectors!

4.1.21. Yes, you can either show it by using the regular 3 steps (Trick 2), or you can show it's the Span of 3 matrices.

4.1.22. Yes, use the regular 3 steps of showing that something is a subspace!

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4.1.24.

(a) **T** (this is important to remember!!! A vector isn't a list of numbers any more, it could be anything, even a function!)
(b) **T**
(c) **T** (of itself!)
(d) **F** (careful, a subspace also has to be a subset! Here $\mathbb{R}^2$ is not a subset of $\mathbb{R}^3$. That is, a list of numbers with 2 entries is not the same as a list of numbers with 3 entries)
(e) **T** (again, the textbook might give you a different answer, but I agree that this is weirdly phrased! What they mean is: If $\mathbf{u}, \mathbf{v}$ is in H, then $\mathbf{u} + \mathbf{v}$ is in H).

4.1.32. This is a bit tricky! Remember that $H \cap K$ is the set of vectors that is both in H and in K. Here's the proof that $H \cap K$ is closed under addition (hopefully that'll inspire you to do the rest):

Suppose $\mathbf{u}$ and $\mathbf{v}$ are in $H \cap K$. Then $\mathbf{u}$ and $\mathbf{v}$ are in H, so is $\mathbf{u} + \mathbf{v}$ (since H is a subspace). Also, since $\mathbf{u}$ and $\mathbf{v}$ are in K, so is $\mathbf{u} + \mathbf{v}$ (since K is a subspace). Hence $\mathbf{u} + \mathbf{v}$ is both in H and K, hence $\mathbf{u} + \mathbf{v}$ is in $H \cap K$.

As for the fact that the union of two subspaces is not a subspace, take H to be the $x-$axis, and K to be the $y-$ axis in $\mathbb{R}^2$.

SECTION 4.3: LINEARLY INDEPENDENT SETS, BASES

Remember that a basis is a linearly independent set which spans the whole space! Equivalently, as set is a basis if the corresponding matrix A is invertible.

4.3.1. Yes (you can show directly that it is linearly independent. Since there are 3 vectors and $dim(\mathbb{R}^3) = 3$, by a theorem in Friday we know that it also spans $\mathbb{R}^3$)

4.3.5. No (lin. dep. because contains the **0** vector, does span $\mathbb{R}^3$ though)

4.3.6. No (even though it's linearly independent, it doesn't span $\mathbb{R}^3$ because $dim(\mathbb{R}^3) = 3$ and there are only 2 vectors)

4.3.11. The system the book talks about is:

$$\begin{cases} x + 2y + z = 0 \\ y = y \\ z = z \end{cases}$$

In other words, you add the two trivial equations $y = y$ and $z = z$. Then solve for x in terms of y and write your answer in parametric vector form

4.3.15. IGNORE THIS PROBLEM, unless you enjoy being tortured :) We will learn a better technique of doing this problem in section 4.6.

4.3.21.

(a) **F** (consider the $\mathbf{0}$-vector)
(b) **F** (no, the set could be linearly dependent!)
(c) **T** (yes, by IMT)
(d) **F** (as *small* as possible, i.e. no redundant vectors)
(e) Ignore

4.3.22.

(a) **F** (might not span H!)
(b) **T** (this is the spanning set theorem, theorem 5b)
(c) **T**
(d) Ignore
(e) Ignore

4.3.33. Suppose $a\mathbf{p}_1 + b\mathbf{p}_2 = \mathbf{0}$, where $\mathbf{0}$ is the zero-polynomial. Then either identify coefficients (like in Math 1B), or differentiate, to find out that $a = b = 0$.

4.3.34. $\mathbf{p_3} = \mathbf{p_1} + \mathbf{p_2}$, so we can eliminate $\mathbf{p_2}$ from the span in the problem! However, $\mathbf{p_1}$ and $\mathbf{p_2}$ are linearly independent (show this! same technique as 33), hence they form a basis for that span.

SECTION 4.4: COORDINATE SYSTEMS

Remember: It's easier to figure out $\mathbf{x}$ once we know $[\mathbf{x}]_{\mathcal{B}}$ than the reverse. Also, remember that the change of coordinates matrix is just the matrix whose columns are the elements in $\mathcal{B}$.

It takes a code as its input and tells you which vector corresponds to that code. Its inverse matrix does what you usually want: It produces the coordinates of $\mathbf{x}$

4.4.1. This just means $\mathbf{x}$ is 5 times the first vector + 3 times the second vector.

4.4.5. Figure out for which coefficients x_1, x_2 we have $x_1\mathbf{b_1} + x_2\mathbf{b_2} = \mathbf{x}$. For this, just solve a system of equations for $\mathbf{x_1}$ and $\mathbf{x_2}$.

4.4.9. Its just the matrix whose columns are the two vectors!

4.4.11. Use the formula: $\mathbf{x} = P_{\mathcal{B}}\,[\mathbf{x}]_{\mathcal{B}}$, where $P_{\mathcal{B}}$ is the change-of-coordinates matrix!

4.4.13. For this, first identify the polynomials with a vector, i.e. $1 + t^2$ becomes $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $1 + 4t + 7t^2$ becomes $\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$. Then find the change-of-coordinates matrix, and use the formula: $\mathbf{p} = P_{\mathcal{B}}\,[\mathbf{p}]_{\mathcal{B}}$ and solve for $[\mathbf{p}]_{\mathcal{B}}$. **Notice how useful this identification is!**

4.4.15.

(a) T
(b) F (it's the opposite: $\mathbf{x} = P_{\mathcal{B}} [\mathbf{x}]_{\mathcal{B}}$, P takes a code as its input and spits out a vector, just like a supermarket scanner!)
(c) Ignore

4.4.16.

(a) T (yes, because for example, $\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, so the code/coefficients of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is just $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$. **DO NOT WRITE DOWN THIS EXAMPLE ON YOUR HW**, it's just for information purposes)
(b) F (it's the opposite: it's the correspondence $\mathbf{x} \longmapsto [\mathbf{x}]_{\mathcal{B}}$ that's the coordinate mapping / producing the code mapping)
(c) Ignore

4.4.27. In other words, is the following set linearly independent or not?

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$$

Again, **notice the usefulness of identifying a vector with its code!!!**